

**LOWER BOUNDS TO HAMMETT  $\sigma$  CONSTANTS FOR  
*meta* AND *para* NORMAL SUBSTITUENTS**João Carlos R. REIS<sup>a,\*</sup>, Manuel A. P. SEGURADO<sup>b</sup> and Jaime D. Gomes de OLIVEIRA<sup>c</sup><sup>a</sup> Departamento de Química e Bioquímica, Centro de Electroquímica e Cinética,  
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We dedicate this paper to Professor Otto Exner on the occasion of his 75th birthday in recognition of his outstanding contributions to the areas of physical organic chemistry and chemometrics.

Twenty-four pairs of *meta* and *para* Hammett  $\sigma$  constants recommended by IUPAC for normal substituents, *i.e.* dipolar groups without a lone pair of electrons (and with a full octet) in the atom next to the aromatic ring, were analysed with respect to their *meta-para* interrelationship. In terms of a previous hyperbolic model, the *para/meta* ratio of the universal electric effect is found to be 0.964 with a standard error of 0.028 estimated by Monte Carlo simulation. This value for benzoic acid derivatives supports the view that the universal electric effect, which is proposed to be termed the Electra effect, is transmitted through space. For normal substituents, it is demonstrated that the hyperbolic model predicts lower bounding values of  $\sigma_m$  and  $\sigma_p$  constants. It is proposed that organometallic substituents in which a metallic atom is bounded to the  $\alpha$  carbon should not be considered as normal substituents. It is asserted that no normal substituent should exist for which either  $\sigma_m$  is less than  $-0.1$  or  $\sigma_p$  is less than  $-0.3$  in the Hammett  $\sigma$  scale.

**Key words:** *meta/para* Ratio; Substituent constants; Substituent effects; Through-space interactions; Hammett equation; Hyperbolic model; Chemometrics.

The *meta-para* interrelationship is one of the many themes in physical organic chemistry to which Exner has contributed<sup>1</sup>. The concept of a normal substituent was clearly stated by Exner as being a "dipolar substituent without a lone pair of electrons (and with a full electron octet) in the  $\alpha$  position" (ref.<sup>2</sup>), *i.e.* in the atom adjacent to the aromatic ring. On the other hand, critically evaluated substituent constants are an important tool in correlation analysis. Therefore, the recently initiated compilation<sup>3</sup> of

IUPAC recommended values for the Hammett  $\sigma$  scale prompted us to analyse these values in terms of the correlations between *meta* and *para*  $\sigma$  constants for normal substituents.

Presently, three models are available for this correlation: The Exner equation<sup>1,2,4,5</sup>, which expresses a direct proportionality law, the bundle of straight lines due to Pytela<sup>6</sup>, and our hyperbolic model<sup>7</sup>. We have outlined<sup>7</sup> other historically interesting approaches. Since the Exner equation applies only to normal substituents, no extended equation for substituent effects could be based on it. However, the more general approach by Pytela has led to the Alternative Interpretation of Substituent Effects (AISE)<sup>8</sup>, and our hyperbolic model to tetralinear extensions<sup>9</sup> of the Hammett and Yukawa–Tsunoo equations.

A common feature to Exner's and Pytela's approaches is that possible values of  $\sigma_m$  and  $\sigma_p$  constants for normal substituents are unbounded. In other words, provided that they conform to these linear models, no upper or lower values are imposed. In contrast, if the hyperbolic model is a correct phenomenological theory of the *meta*–*para* interrelationship in benzene derivatives, then we can predict lower boundary values of  $\sigma_m$  and  $\sigma_p$  constants for normal substituents. The main aim of the present paper is to demonstrate this assertion.

#### THEORETICAL AND CALCULATIONS

The hyperbolic model<sup>7</sup> describes substituent effects<sup>9</sup> free from direct or through-resonance effects. Since the Hammett  $\sigma$  scale is referred to the benzoic acid reactivity, special substituents, *i.e.*, those bearing a lone electron pair (or an incomplete electron octet) in the atom adjacent to the benzene ring, were excluded from this analysis. Pairs of  $\sigma_m$  and  $\sigma_p$  values for 24 normal substituents were taken from Shorter's compilation<sup>3</sup>. For convenience, the data used are collected in Table I where substituents are ordered by increasing  $\sigma_p$  values. In this list examples can be found of normal substituents where atoms of carbon, nitrogen, sulfur and phosphorus are in the  $\alpha$  position.

In terms of the hyperbolic model, the correlation equation in a plot of  $\sigma_p$  against the corresponding  $\sigma_p/\sigma_m$  ratio is Eq. (1) (ref.<sup>7</sup>),

$$\sigma_p = \frac{\gamma \sigma_p / \sigma_m + \varepsilon_n}{\sigma_p / \sigma_m - \lambda} \quad (1)$$

TABLE I  
IUPAC recommended Hammett  $\sigma$  constants for normal substituents used in this work and their limits of uncertainty<sup>a</sup>

Substituent	$\sigma_m$	$\pm$ limit of uncertainty	$\sigma_p$	$\pm$ limit of uncertainty
Cyclopropyl	-0.070	0.02	-0.210	0.02
Me	-0.058	0.01	-0.165	0.01
<i>tert</i> -Bu	-0.070	0.02	-0.163	0.04
Isopropyl	-0.080	0.02	-0.151	0.02
Et	-0.060	0.02	-0.150	0.02
CH <sub>2</sub> Ph	-0.079	0.04	-0.106	0.04
CH <sub>2</sub> OMe	0.020	0.04	0.026	0.04
CH <sub>2</sub> I	0.073	0.04	0.086	0.04
CH <sub>2</sub> Cl	0.086	0.04	0.119	0.04
CH <sub>2</sub> Br	0.106	0.04	0.119	0.04
CH <sub>2</sub> CN	0.152	0.04	0.172	0.04
C $\equiv$ CH	0.198	0.04	0.224	0.04
CO <sub>2</sub> Me	0.330	0.02	0.445	0.02
CO <sub>2</sub> Et	0.363	0.04	0.449	0.04
COMe	0.368	0.02	0.486	0.02
POBu <sub>2</sub>	0.376	0.04	0.515	0.04
CF <sub>3</sub>	0.435	0.03	0.528	0.04
PO(OMe) <sub>2</sub>	0.420	0.03	0.550	0.03
POPh <sub>2</sub>	0.440	0.04	0.580	0.04
SO <sub>2</sub> NH <sub>2</sub>	0.530	0.02	0.580	0.02
CN	0.623	0.02	0.668	0.02
SF <sub>5</sub>	0.613	0.04	0.693	0.04
SO <sub>2</sub> Me	0.675	0.02	0.717	0.02
NO <sub>2</sub>	0.734	0.02	0.777	0.01

<sup>a</sup> Non-rounded values taken from Shorter<sup>3</sup>. See ref.<sup>3</sup> for the original references.

Equation (1), which describes a rectangular hyperbola, has a horizontal asymptote defined by  $\sigma_p = \gamma$  and a vertical asymptote given by  $\sigma_p/\sigma_m = \lambda$ . The adjustable parameters  $\lambda$ ,  $\gamma$  and  $\varepsilon_n$  have been discussed<sup>7,9</sup>. Here it suffices to recall that  $\lambda$  gives the *para/meta* ratio for the universal electric effect<sup>10</sup> or non-mesomeric substituent effect,  $\gamma$  may be regarded as the absolute Hammett reaction constant, and the quantity  $\varepsilon_n + 2\gamma\lambda$  accounts for statistical errors.

A non-linear procedure is needed for the fit of experimental data to Eq. (1). Moreover, because both variables are subjected to errors, we have used an optimization algorithm leading to the least sum of rectangle areas<sup>7,9</sup>. Exner and co-workers<sup>5,11</sup> have a different approach to the problem of regressions with errors in both coordinates. They propose using the squared perpendicular distances from the regression line as the least-squares condition. Although this method is easily implemented when the regression curve is a straight line, it becomes complex in the general case of curve fitting. Therefore, we used the non-linear least-rectangles method. Also, since only limits of uncertainty are known for the IUPAC recommended sigma constants<sup>3</sup>, we preferred to use the non-weighted version of our non-linear least-rectangles program to fit data in Table I to Eq. (1). Standard errors in the fitting parameters were estimated by a Monte Carlo simulation method<sup>12</sup>. For this purpose, sets of pseudo  $\sigma$  constants were randomly generated and then fitted to Eq. (1). For these synthetic data, we fixed a gaussian standard error equal to half the limit of uncertainty given in Table I. Monte Carlo error estimates are based on 1 000 simulations of which 170 did not converge because for at least one point, the resulting rectangle area was negative.

## RESULTS AND DISCUSSION

### *Analysis of the Hammett $\sigma$ Scale*

Best parameter values for fitting the 24 pairs of  $\sigma_m$  and  $\sigma_p$  constants in Table I to Eq. (1) are given in Table II together with associated standard errors.

Values in Table II may be compared with those for the unified  $\sigma$  zero scale<sup>13</sup>, which were found to be<sup>9b</sup>  $\lambda^0 = 0.961(0.005)$ ,  $\gamma^0 = -0.225(0.008)$  and  $\varepsilon_n^0 = 0.343(0.001)$ . The latter values are based on a larger set<sup>7</sup> comprising 10 special and 26 normal substituents of which only 12 are common to both sets. Since different values for a given parameter are at least approximately within their combined standard errors, we conclude that both scales are consistent with each other.

With respect to the recent reappraisal of the universal electric effect<sup>10</sup>, which might be termed the Electra effect, we point out that the comparison between experimental and theoretically calculated  $\lambda$  values provides a definite answer<sup>9</sup> to the mode of transmission problem. In particular, most of the difficulties associated with the calculation of absolute electric effects<sup>10</sup> disappear when ratios *para* to *meta* are considered. Thus the present finding of a  $\lambda$  value (Table II) for benzoic acid derivatives (Table I) in accord<sup>9a</sup> with the *para/meta* ratio of electric field effects is noteworthy. Consequently, Hammett  $\sigma$  constants for normal substituents should express a blend of the universal electrical or Electra effect stabilized through space by Coulomb interactions<sup>10c</sup> with a mesomeric electric effect transmitted through the  $\pi$  network.

Solving Eq. (1) for  $\sigma_m$  gives Eq. (2),

$$\sigma_m = \frac{\sigma_p(\sigma_p - \gamma)}{\lambda\sigma_p + \varepsilon_n}. \quad (2)$$

It should be noted that the hyperbolic model does not allow to express  $\sigma_p$  as an explicit rational function of  $\sigma_m$ . Consequently, in terms of this model  $\sigma_p$  is not a single-valued function of  $\sigma_m$ . Nevertheless, in a plot of  $\sigma_m$  against  $\sigma_p$  Eq. (2) represents a hyperbola with a vertical asymptote given by  $\sigma_p = -\varepsilon_n/\lambda$  and an oblique asymptote given by  $\sigma_m = \sigma_p/\lambda - \gamma/\lambda - \varepsilon_n/\lambda^2$ . Since its parameters are known (Table II), both branches of the hyperbola defined by Eq. (2) are drawn in Fig. 1 where data points are from Table I. Conformity of the IUPAC recommended<sup>3</sup> Hammett  $\sigma$  scale for normal substituents to the hyperbolic model<sup>7</sup> may be appreciated.

Most of the substituents for which both  $\sigma_m$  and  $\sigma_p$  constants are tabulated by Shorter<sup>3</sup> were considered in this analysis. There are, however, some exceptions warranting further discussion. Thus the hydrogen had to be excluded because it leads to an indeterminate value for the ratio  $\sigma_p/\sigma_m$ .

TABLE II  
Best parameter values in Eq. (1) and their standard errors (s) to fit data in Table I

$\lambda$	$\gamma$	$\varepsilon_n$
0.964(0.028)	-0.257(0.021)	0.365(0.040)

Nevertheless, this point lies on the rectangular hyperbola<sup>7</sup>, irrespective of its parameter values. Forcing a *meta-para* model for normal substituents through the point corresponding to the unsubstituted compound, is a feature also adopted by Exner and co-workers<sup>5</sup>. Another case is the vinyl group. There is no doubt in classifying the vinyl group as a normal substituent<sup>2,5</sup>. It is placed in the second division<sup>3a</sup> of the Hammett  $\sigma$  scale with  $\sigma_m = 0.08 \pm 0.03$  and  $\sigma_p = -0.08 \pm 0.03$ . However, since for a normal substituent  $\sigma_m$  and  $\sigma_p$  should have the same sign in the  $\sigma$  zero scale, we believe that there is a through-resonance contribution to the tabulated  $\sigma_p$  value for the vinyl group<sup>3a</sup>. Lastly, the case of the  $\text{SO}_2\text{CF}_3$  substituent for which<sup>3b</sup>  $\sigma_m = 0.80 \pm 0.04$  and  $\sigma_p = 0.96 \pm 0.04$ . As noted by Exner and co-workers<sup>1,4,5</sup>, polyfluorinated substituents tend to deviate from other normal substituents in *meta-para* correlations. To a lesser extent, this deviation is also observed in Fig. 1 for carbonyl and phosphonyl substituents. In these cases, we suspect that deviations may be due to a preferential conformation of the *meta* derivative leading to a higher individual  $\lambda$  value. Then a moderate distortion in plots should serve to remind us of the inherently coarse-grained nature of any *meta-para* model. Despite these limitations, the hyperbolic model may also be appraised by its consequences in establishing lower bounding values for Hammett  $\sigma$  constants.

### Lower Bound to $\sigma_m$ Constants

Inspection of Fig. 1 reveals that  $\sigma_m$  values lying between the upper and the lower branches cannot be described by the hyperbolic model. These particular values can be inferred from the roots of  $d\sigma_m/d\sigma_p = 0$ . This condition is

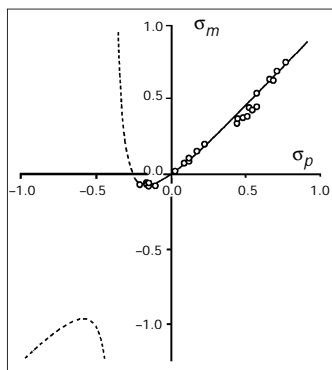


FIG. 1

The *meta* versus *para* relationship with Hammett  $\sigma$  constants for 24 normal substituents. Data points from Table I and lines drawn for Eq. (2) with parameter values in Table II. Sections in dashed line should be void

readily obtained from the differential form of Eq. (2), which is given by Eq. (3),

$$\frac{d\sigma_m}{d\sigma_p} = \frac{\lambda\sigma_p^2 + 2\varepsilon_n\sigma_p - \gamma\varepsilon_n}{(\lambda\sigma_p + \varepsilon_n)^2}. \quad (3)$$

Therefore, it follows from Eq. (3) that  $\sigma_m$  is an extreme when  $\sigma_p$  is a root of Eq. (4),

$$\lambda\sigma_p^2 + 2\varepsilon_n\sigma_p - \gamma\varepsilon_n = 0. \quad (4)$$

Using the parameter values in Table II, the roots of Eq. (4) are found to be  $\sigma_p[1] = -0.593$  and  $\sigma_p[2] = -0.164$ . Lastly, inserting these values into Eq. (2) leads to the extreme values  $\sigma_m[1] = -0.074$  and  $\sigma_m[2] = -0.963$ , respectively. Evidently, the former is the lower bound to  $\sigma_m$  values in the upper branch of hyperbola shown in Fig. 1. Hence  $\sigma_m$  values between  $-0.1$  and  $-0.9$  are apparently forbidden by the hyperbolic model. Clearly, the latter value is sufficiently low to preclude the possibility of even lower  $\sigma_m$  values. Consequently, we assert that no normal substituent should exist for which  $\sigma_m$  is less than  $-0.1$ .

#### *Lower Bound to $\sigma_p$ Constants*

The search for a lower bound to  $\sigma_p$  constants of normal substituents requires a different approach. Thus we resort to the dual electronic model for  $\sigma$  constants<sup>14</sup>. Following Hine<sup>15</sup>, we write Eqs (5) and (6):

$$\sigma_m = \rho_M M + \rho_N N, \quad (5)$$

$$\sigma_p = M + N. \quad (6)$$

In Eqs (5) and (6),  $M$  stands for mesomeric effects,  $N$  for non-mesomeric effects (in fact, universal electric or Electra effects), and  $\rho_M$  and  $\rho_N$  are the respective transmission coefficients from the *meta* position. This separation of electronic effects was previously discussed<sup>7</sup> in terms of the hyperbolic

model. Then we demonstrated<sup>7</sup> that  $\rho_N = 1/\lambda$ ,  $0 \leq \rho_M \leq 1/2\lambda$  and that  $M$  values are bounded as in Eq. (7),

$$\frac{\gamma}{1 - \lambda\rho_M} \leq M \leq -\frac{\gamma}{1 - \lambda\rho_M}. \quad (7)$$

Additionally, Eq. (8) linking  $M$  and  $N$  values for normal substituents was derived<sup>7</sup>,

$$M \left[ M - \frac{(1 - 2\lambda\rho_M)\gamma}{1 - \lambda\rho_M} \right] / \left( M + \frac{\gamma}{1 - \lambda\rho_M} \right) = -N. \quad (8)$$

We remark that Eq. (8) is the corrected form of Eq. (14) in ref.<sup>7</sup> where a misprint occurred. From the differential form of Eq. (8), or from Fig. 4 in ref.<sup>7</sup>, it can be shown that in the region where  $M$  is negative,  $M$  decreases faster than  $N$  increases, *i.e.*  $-dM/dN > 1$ . In view of Eq. (6), then the minimum in  $\sigma_p$  is attained for the lowest  $M$  value, which is found from Eq. (7) to be  $\gamma/(1 - \lambda\rho_M)$ . Introduction of this value in Eq. (8) leads to the corresponding  $N$  value, which is  $-\lambda\rho_M\gamma/(1 - \lambda\rho_M)$ . Hence the sum  $M + N$  (see Eq. (6)) is found to be  $\gamma$ . In other words, the lower bound to  $\sigma_p$  values for normal substituents is  $\gamma = -0.26$  to which corresponds a zero value for  $\sigma_m$  (see Eq. (2)).

### Analysis of Lower Bounds

Following a suggestion by a Referee, we tested the above lower bounds with the survey of Hammett  $\sigma$  constants for 530 substituents by Hansch *et al.*<sup>16</sup> To avoid discussing border-line cases, we added an uncertainty of  $-0.04$  to our lower limits. Thus we found 23 substituents for which  $\sigma_m < -0.12$  and 26 substituents for which  $\sigma_p < -0.30$ . Upon exclusion of substituents with an incomplete octet or a lone electron pair in the  $\alpha$  atom, charged substituents and fused-ring systems, we were left with only 7 *meta* and 3 *para* sigma values deserving further discussion. In 4 cases of typically normal substituents ( $\text{CH}_2\text{C}(\text{OH})\text{Me}_2$ ,  $\text{CH}_2\text{SiMe}_3$ ,  $\text{CH}_2\text{CH}_2\text{SiMe}_3$  and ferrocenyl) tabled  $\sigma_m$  values<sup>16</sup> differ from our lower bound value by twice an uncertainty of  $-0.04$ . We suspect that these  $\sigma_m$  values are only approximate although they fall within the uncertainty of  $\pm 0.1$  suggested by McDaniel and Brown<sup>17</sup> for secondary sigma values.

Particularly interesting are the remaining 6 cases. The latter are for 3 organometallic substituents (entries 401, 469 and 489 in Table I of ref.<sup>16</sup>)



for which  $\sigma_m = -0.14$ ,  $\sigma_p = -0.44$  [ $\text{CH}_2\text{Mn}(\text{CO})_5$ ];  $\sigma_m = -0.26$ ,  $\sigma_p = -0.49$  [ $\text{CH}_2\text{Fe}(\text{CO})_2(\pi\text{-C}_5\text{H}_5)$ ] and  $\sigma_m = -0.21$ ,  $\sigma_p = -0.45$  [ $\text{CH}_2\text{Mo}(\text{CO})_3(\pi\text{-C}_5\text{H}_5)$ ]. Very large  $\sigma_p$  values together with  $\sigma_p/\sigma_m$  ratios between 1.9 and 3.1 are a strong indication for direct resonance (delocalized) electrical effects in these substituents. Given the special nature of carbon-metal bonds, there is a case for extending Exner's definition<sup>2</sup> of a normal substituent by excluding substituents in which a metallic atom is bonded to the  $\alpha$  carbon.

In conclusion, on the basis of the hyperbolic model<sup>7</sup> for the *meta-para* interrelationship in benzene derivatives applied to IUPAC recommended values<sup>3</sup>, we have demonstrated that Hammett  $\sigma$  constants for normal substituents have lower bounding values. These are found to be  $-0.08$  for  $\sigma_m$  and  $-0.26$  for  $\sigma_p$  constants, so that the dashed lines in Fig. 1 should be empty. Consequently, a finding of normal substituents with  $\sigma$  constants distinctly below these bounding values would jeopardise our hyperbolic model.

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